NEW DECISION RULES IN CONSTRUCTION MODELS FOR VEHICLE ROUTING PROBLEMS WITH TIME WINDOWS

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Presentation outline

- Problem Analysis
- Model Development
- Results
- Conclusions

1. Problem Analysis

VRPTW characteristics

- Central depot and multiple customers geographically dispersed
- Depot with time window
- Customers with variable time windows
- Variable service times for customers
- Limited capacity for all vehicles
- Limited number of vehicles
- Euclidean distances considered
- Goal: Minimize total distance

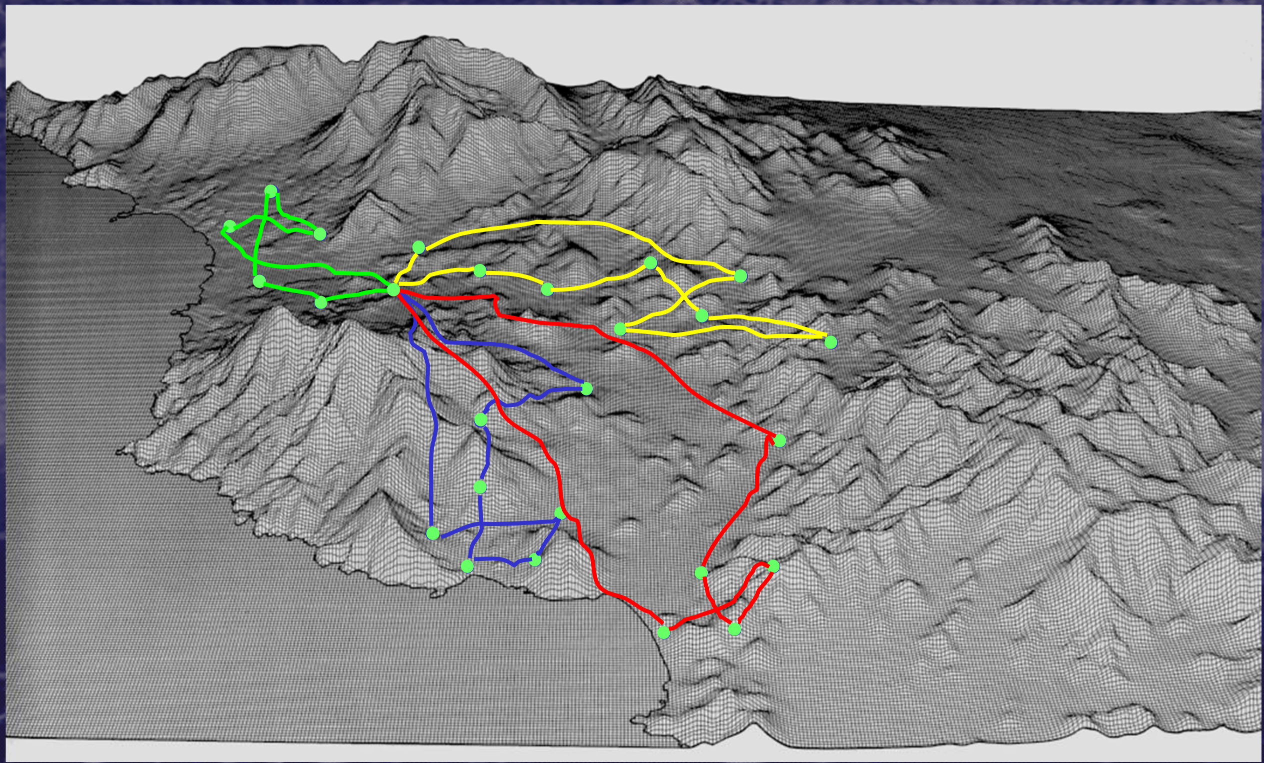
1. Problem Analysis

2. Model development

3. Results

4. Conclusions

The Vehicle Routing Problem with Time Windows



- Open Node
- Closed Node

Mathematical Model for the VRPTW

$$\min \sum_{k\in V}\sum_{i\in N}\sum_{j\in N} c_{ij} x_{ij}^k$$

$$\sum_{k\in V}\sum_{j\in N} x_{ij}^k = 1 \quad \forall i \in N$$

$$\sum_{j\in N} x_{0j}^k = 1 \quad \forall k \in V$$

$$\sum_{i\in N} x_{i0}^k = 1 \quad \forall k \in V$$

$$\sum_{i\in N} x_{ih}^k - \sum_{j\in N} x_{hj}^k = 0 \quad \forall h \in N, \forall k \in V$$

$$\sum_{i\in N} d_i \sum_{j\in N} x_{ij}^k \le q \quad \forall k \in V$$

$$x_{ij}^k (S_i^k + t_{ij} - S_j^k) \le 0 \quad \forall i, j \in N, \forall k \in V$$

$$a_i \le S_i^k \le b_i \quad \forall i \in N, \forall k \in V$$

$$x_{ij}^k = (0,1) \quad i, j \in N, \forall k \in V$$

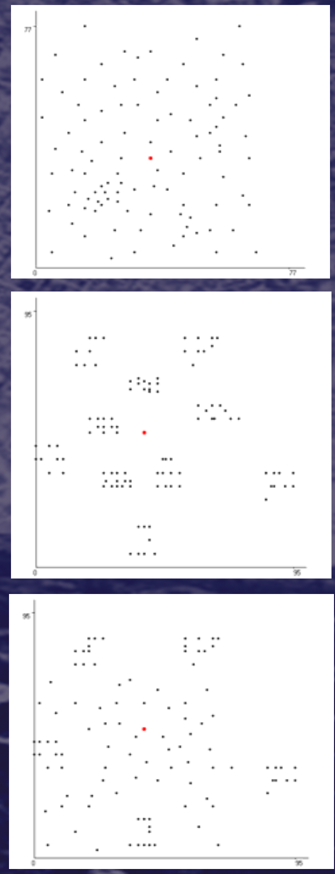
Solution techniques for the VRPTW

- Heuristics
- Construction methods
  - Solomon (1987)
    - Savings methods
    - Gillett and Miller
    - Insertion I1
    - Insertion I2
    - Insertion I3
  - Potvin and Rousseau (1993)
  - Dullaert (2000 and 2001)
  - Ioannou et al (2001)
  - Bramel and Simchi-Levi (1996)

1. **Problem Analysis**
2. Model development
3. Results
4. Conclusions

Solution techniques for the VRPTW

- Improvement Methods
  - Intra-route
  - Inter-route
    - Cyclic transfers
  - Antes and Derigs (1995)
  - Russell (1995)
  - Shaw (1997)
  - Cordone and Wolfer Calvo (1998)
  - Bräysis (2001)
- Meta-heuristics
  - Tabu Search
  - Simulated Annealing
  - Genetic Algorithms
  - Others



VRPTW

Solomon (1987)

2. Model Development

Model characteristics

"Paralell construction of the routes in different phases, following different decision rules"

- Deterministic construction model
- Parameters inclusion
- Temporal and spatial criteria
- Basic rules: assignment, addition, single insertion and double insertion

Initial data

r103 - Bloc de notas

Archivo Edición Buscar Ayuda

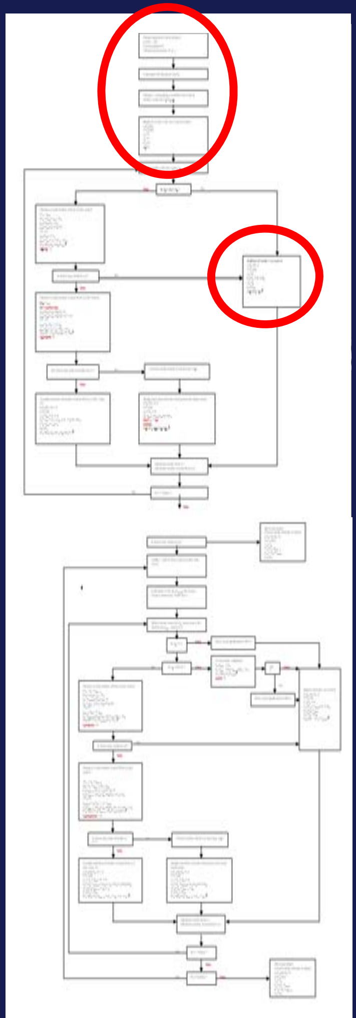
R103

VEHICLE

NUMBER	CAPACITY
25	200

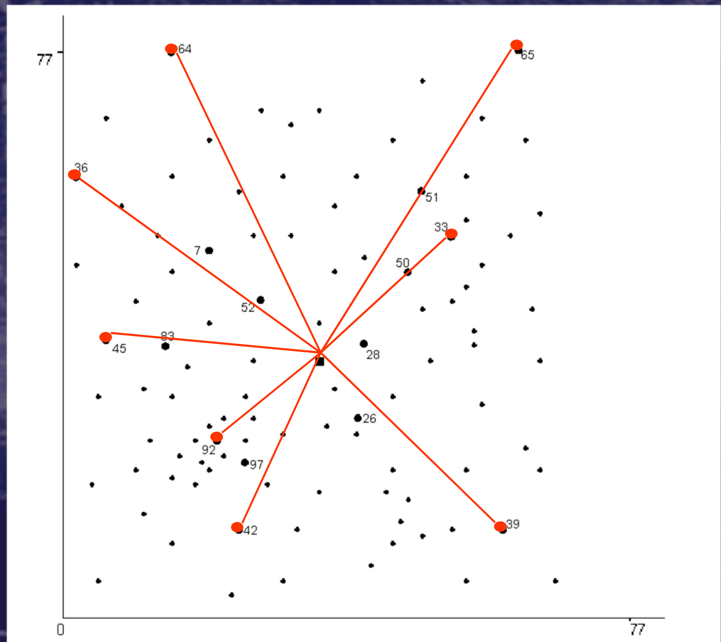
CUSTOMER

CUST NO.	XCOORD.	YCOORD.	DEMAND	READY TIME	DUE DATE	SERVICE TIME
0	35	35	0	0	230	0
1	41	49	10	0	204	10
2	35	17	7	0	202	10
3	55	45	13	0	197	10
4	55	20	19	149	159	10
5	15	30	26	0	199	10
6	25	30	3	99	109	10
7	20	50	5	0	198	10
8	10	43	9	95	105	10
9	55	60	16	97	107	10
10	30	60	16	124	134	10
11	20	65	12	67	77	10
12	50	35	19	0	205	10
13	30	25	23	159	169	10
14	15	10	20	0	187	10
15	30	5	8	61	71	10
16	10	20	19	0	190	10
17	5	30	2	157	167	10
18	20	40	12	0	204	10
19	15	60	17	0	187	10
20	45	65	9	0	188	10
21	45	20	11	0	201	10
22	45	10	18	97	107	10
23	55	5	29	68	78	10
24	65	35	3	0	190	10
25	65	20	6	172	182	10



Initial assignment rule

R103 *R=8, β=1.2 y γ=2*



tc_i

t_{0i}

$h_i = tc_i - t_{0i}$

Direct addition

$LR_k = [0, i]$

$CR_k = [i]$

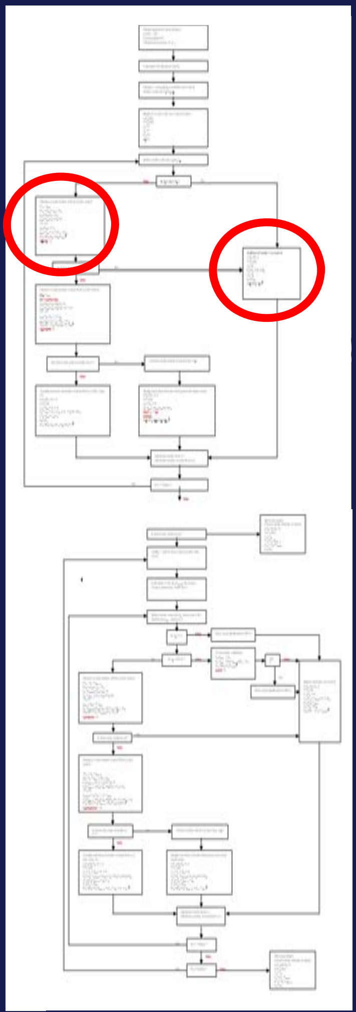
$c_k = d_i$

$tr_k = t_{0i} + ts_i + te_{ki}$

$D_k = t_{0i}$

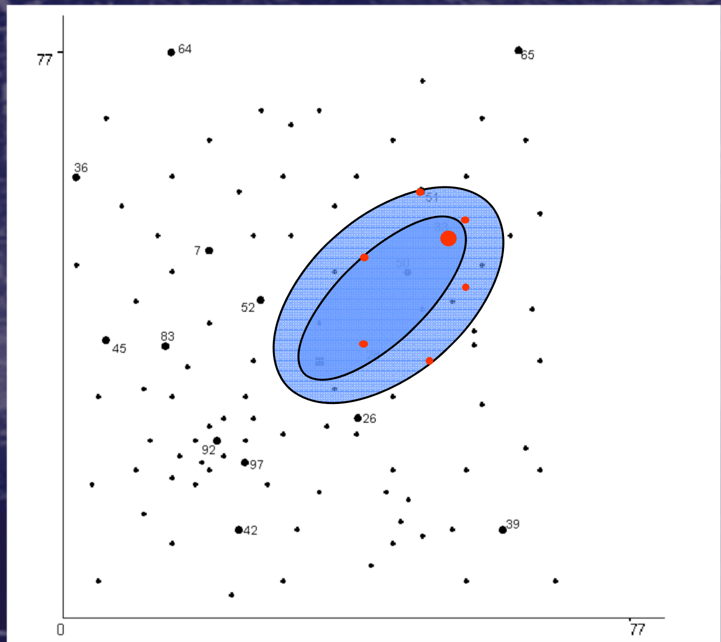
$E_k = te_{ki}$

$te_{ki} = [ta_i - (t_{0i})]^+$



Time gap analysis for possible insertions

R103 *R=8, β=1.2 y γ=2* *min $h_i = tc_i - t_{0i}$*



L2 creation

$ta_m \le t_{om}$

$ta_m + ts_m + t_{mi} \le tc_i$

$t_{0m} + ts_m + t_{mi} \le tc_i$

$t_{0m} + ts_m + t_{mi} + nh_i = tc_i$

$nh_i \ge 0$

$t_{0m} + t_{mi} \le \beta \cdot t_{0i}$

$t_{0m} + t_{mi} + t_{i0} + te_{ki} \le tc_0$

$te_{ki} = [ta_i - (t_{0m} + ts_m + t_{mi})]^+$

$d_m + d_i \le q$

Direct addition

$LR_k = [0, i]$

$CR_k = [i]$

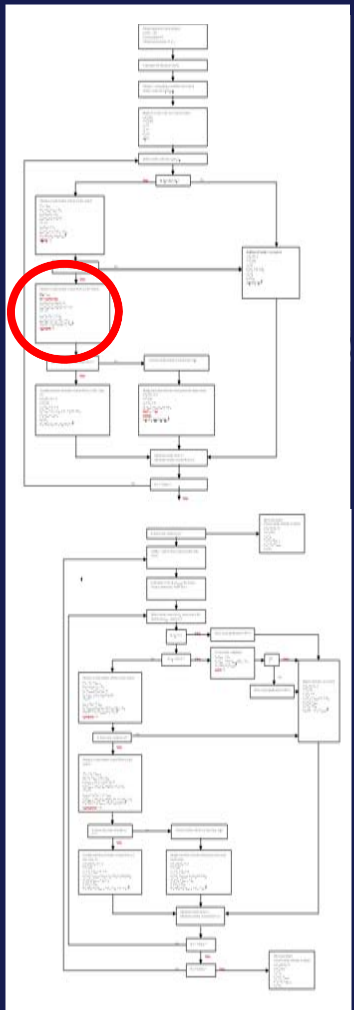
$c_k = d_i$

$tr_k = t_{0i} + ts_i + te_{ki}$

$D_k = t_{0i}$

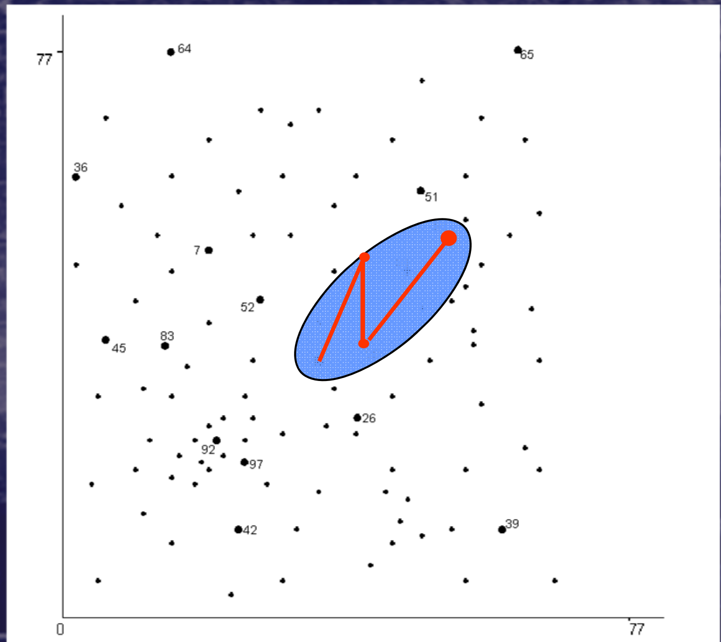
$E_k = te_{ki}$

$te_{ki} = [ta_i - (t_{0i})]^+$



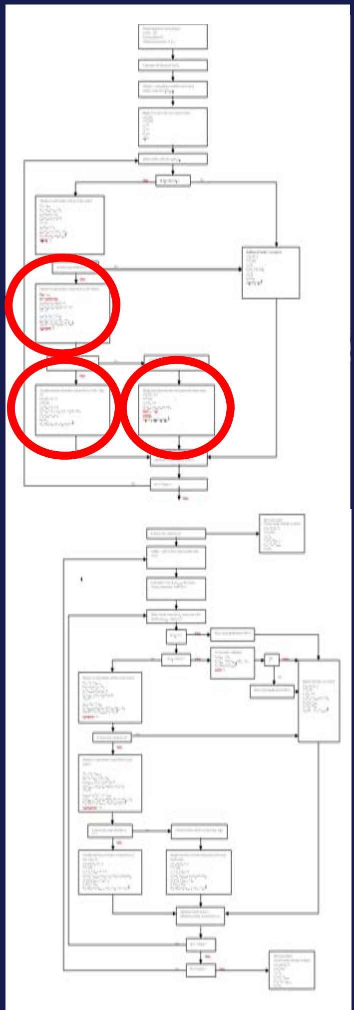
Double Insertion Requirements

R103 *R=8, β=1.2* y *γ=2*



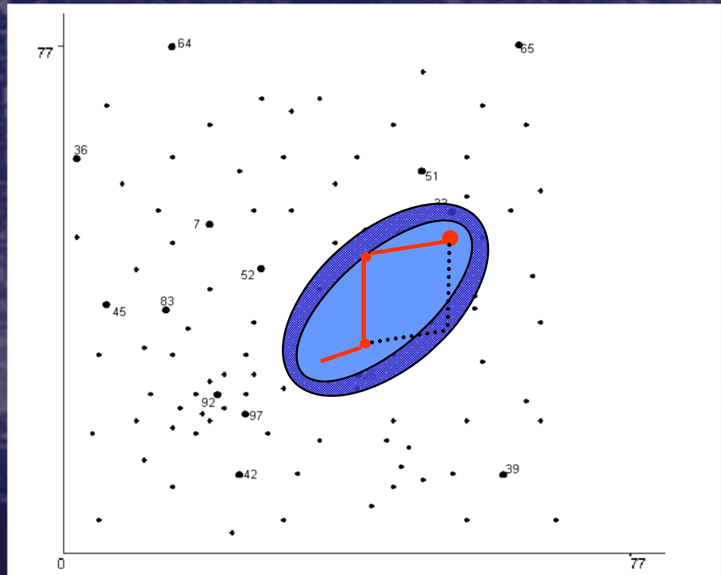
L3 creation

$$ta_m \le t_{0m}$$
$$ta_l \le t_{om} + ts_m + t_{ml}$$
$$t_{0m} + ts_m + t_{ml} + ts_l + t_{li} \le tc_i$$
$$t_{0m} + ts_m + t_{ml} + ts_l + t_{li} + nh'_i = tc_i$$
$$nh'_i \ge 0$$
$$t_{0m} + t_{ml} + t_{li} \le \gamma \cdot \beta \cdot t_{0i}$$
$$t_{0m} + t_{ml} + t_{li} + t_{i0} + te_{ki} \le tc_0$$
$$te_{ki} = [ta_i - (t_{0m} + ts_m + t_{ml} + ts_l + t_{li})]^+$$
$$d_m + d_l + d_i \le q$$



Double or Single insertion procedures

R103 *R=8, β=1.2* y *γ=2*



L3 Creation

$ta_m \le t_{0m}$

$ta_l \le t_{0m}+ts_m+t_{ml}$

$t_{0m}+ts_m+t_{ml}+ts_l+t_{li} \le tc_i$

$t_{0m}+ts_m+t_{ml}+ts_l+t_{li}+nh'_i = tc_i$

$nh'_i \ge 0$

$t_{0m}+t_{ml}+t_{li} \le \gamma \cdot \beta \cdot t_{0i}$

$t_{0m}+t_{ml}+t_{li}+t_{i0}+te_{ki} \le tc_0$

$te_{ki}=[ta_i-(t_{0m}+ts_m+t_{ml}+ts_l+t_{li})]^+$

$d_m+d_l+d_i \le q$

Double Insertion

$LR_k=[0, m, l, i]$

$CR_k=[i]$

$c_k= d_m + d_l + d_i$

$tr_k= t_{0m} + ts_m + t_{ml} + ts_l + t_{li}+ts_i+te_{ki}$

$D_k= t_{0m} + t_{ml}+ t_{li}$

$E_k=te_{ki}$

$te_{ki}=[ta_i-(t_{0m}+ts_m+t_{ml}+ts_l+t_{li})]^+$

Single Insertion

$LR_k=[0, m, i]$

$CR_k=[i]$

$c_k= d_m + d_i$

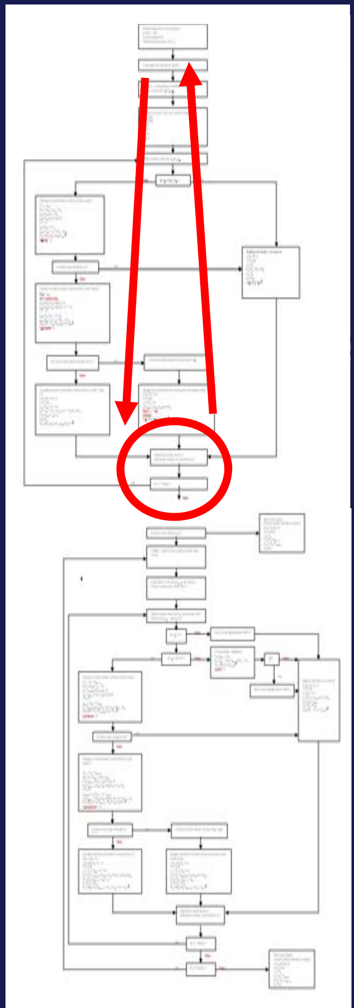
$tr_k=t_{0m} + ts_m+t_{mi}+ts_i+te_{ki}$

$D_k= t_{0m} + t_{mi}$

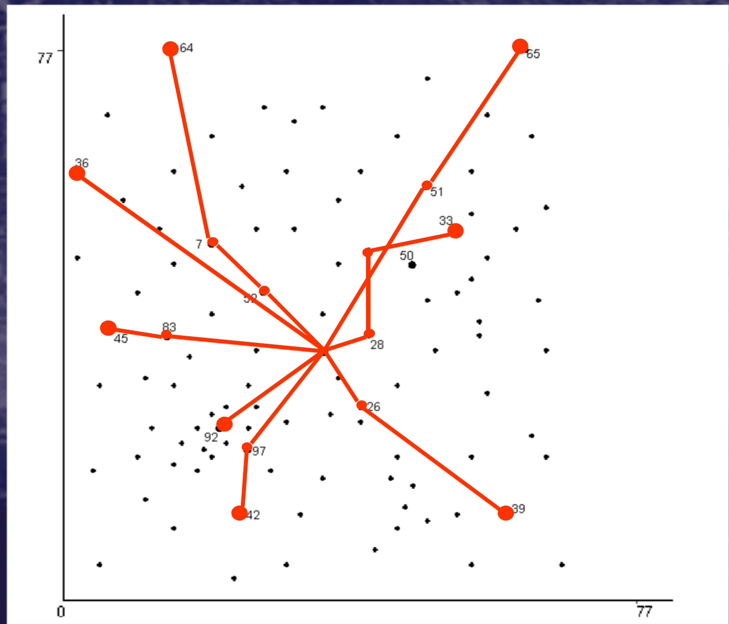
$E_k=te_{ki}$

$te_{ki}=[ta_i-[t_{0m}+ts_m+t_{mi})]^+$

End of first phase



R103 *R=8, β=1.2* y *γ=2*

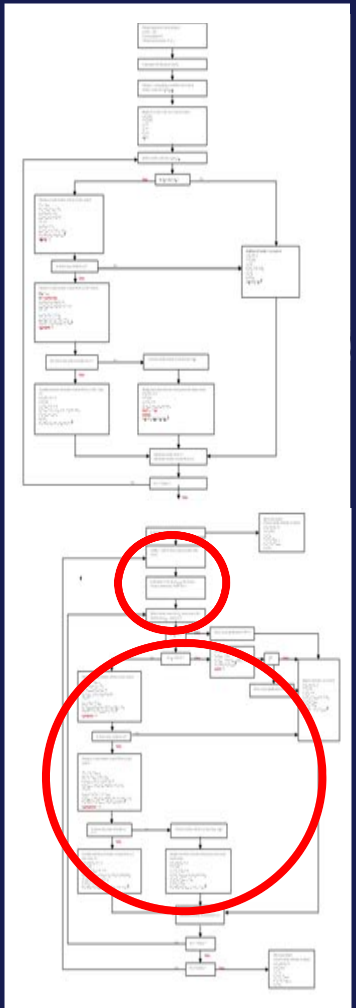


End of first phase

Routes uptdate

New selection of *R* nodes with

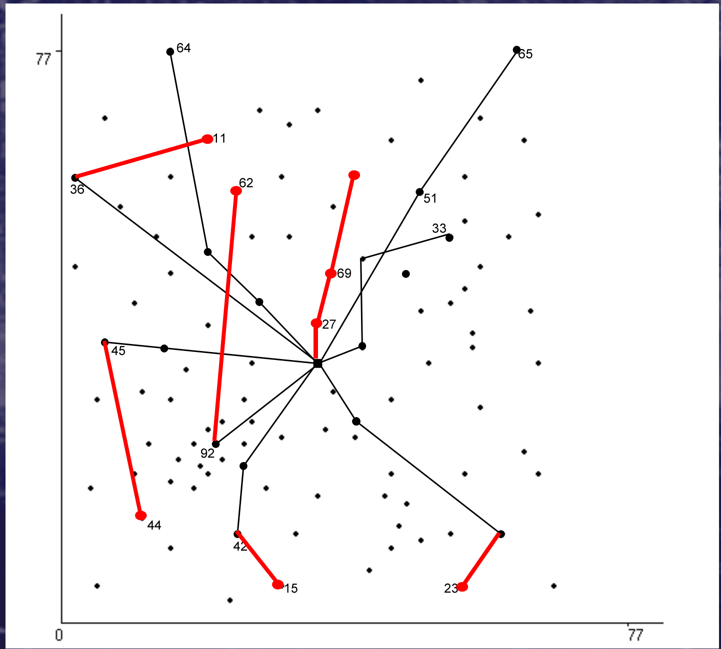
min tc_i



Second and further assignments

R103

R=8, β=1.2 y γ=2



min tc_i

Beginning of second and further phases

One by one all nodes are assigned to the nearest vehicles, given that these arrive on time.

3. Results

Comparison with other construction methods

Method	*R1*		*R2*		*RC1*		*RC2*		*C1*		*C2*	
	Dist	NV	Dist	NV	Dist	NV	Dist	NV	Dist	NV	Dist	NV
Clarke and Wright Method	1499	16,60	-	-	-	-	-	-	976	11,70	-	-
Savings, waiting time limit	1517	15,10	-	-	-	-	-	-	987	10,70	-	-
Solomon I1	1437	13,60	1402	3,30	1597	13,50	1682	3,90	952	10,00	692	3,13
Solomon I2	1639	14,50	1471	3,30	1874	14,20	1798	4,10	1050	10,10	921	3,40
Solomon I3	1652	14,10	1475	3,40	1850	14,00	1816	4,00	1103	10,00	1073	3,50
Nearest Neighbour	1600	14,50	1472	3,40	1800	14,20	1755	3,90	1171	10,20	963	3,50
Gillet and Miller Method	1500	14,60	1449	3,20	1804	14,90	1736	4,00	941	10,00	712	3,00
Potvin and Rousseau (1993)	1509	13,30	1387	3,10	1724	13,40	1651	3,60	1343	10,67	797	3,38
Ioannu et al. (2001)	1370	12,67	1310	3,09	1512	12,50	1483	3,50	865	10,00	662	3,13
Guillén et al. (2004)	1955	26,00	1239	8,00	2247	21,00	1573	11,00	1955	25,00	1485	14,00

4. Conclusions

Conclusions

- The area limitation procedure works well
- If all nodes are subject to time window constraints, the algorithm performs worse
- Time windows are necessary
- Free nodes to be inserted are also necessary
- In type 2 problems, the algorithm performs better, since time gaps are wider
- Node dispersion makes insertions possible
- Solomon results are improved for R2 and RC2 problems

Thank you !

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